

A Space-Frequency Transmitter Diversity Technique for OFDM Systems

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Abstract—A transmitter diversity technique for wireless communications over frequency selective fading channels is presented. The proposed technique utilizes orthogonal frequency division multiplexing (OFDM) to transform a frequency selective fading channel into multiple flat fading subchannels on which space-frequency processing is applied. Simulation results verify that in slow fading environments the proposed space-frequency OFDM (SF-OFDM) transmitter diversity technique has the same performance as a previously reported space-time OFDM (ST-OFDM) transmitter diversity system but shows better performance in the more difficult fast fading environments. Other implementation advantages of SF-OFDM over the ST-OFDM transmitter diversity technique are also discussed.

I. INTRODUCTION

Spatial diversity is a well-known technique for combating the detrimental effects of multipath fading. Traditionally, spatial diversity has been implemented at the receiver end, requiring multiple antennas and RF front-end circuits at the receivers. This multiplicity of receiver hardware is a major drawback, especially for portable receivers where physical size and current drain are important constraints. In recent years, transmitter diversity has received strong interest. The main advantage of transmitter diversity is that diversity gain can be achieved by transmitting from multiple spatially separated antennas without significantly increasing the size or complexity of the receivers.

A number of orthogonal space-time transmitter diversity techniques have been proposed [1-3]. Unfortunately, the large delay spreads in frequency selective fading channels destroy the orthogonality of the received signals, which is critical to the operation of the diversity systems. Consequently, these techniques are often only effective over flat fading channels, such as indoor wireless networks or low data rate systems. Space-time coded OFDM (ST-OFDM) systems [4, 5] have been proposed recently for delay spread channels. In [5], it was shown that OFDM modulation with cyclic prefix can be used to transform frequency selective fading channels into multiple flat fading channels so that orthogonal space-time transmitter diversity can be applied, even for channels with large delay spreads. The use of OFDM also offers the possibility of coding in the frequency

dimension in a form of space-frequency OFDM (SF-OFDM) transmitter diversity, which has also been suggested in [2]. This paper focusses on the implementation and performance evaluation of SF-OFDM transmitter diversity and its possible advantages over ST-OFDM transmitter diversity. Special considerations for the effective implementation of SF-OFDM transmitter diversity are also discussed.

II. OFDM MODULATION WITH CYCLIC PREFIX

OFDM is a popular modulation scheme for high data rate digital communications, especially for channels with large delay spreads [6]. A block diagram of a conventional OFDM communication system is shown in Fig. 1. An OFDM communication system can be considered as a block or vector transmission system. Let $X(m)$ denote the input serial data symbols with symbol duration T_s . The serial to parallel converter collects N serial data symbols into a data vector $X(n) = [X(nN) X(nN+1) \dots X(nN+N-1)]^T$, which has a block duration of NT_s . An even block size N is assumed throughout the paper. In fact, N is often chosen to be powers of two to take advantage of the efficiency of the fast Fourier transform (FFT) in the modulation and demodulation processes. Let $X_k(n)$ denote the k -th forward polyphase component of the serial data symbols, i.e., $X_k(n) = X(nN+k)$ for $k=0,1,\dots,N-1$. $X_k(n)$ can be viewed as the data symbol to be transmitted on the k -th subcarrier during the block instant n . The data vector $X(n)$ can then be expressed in polyphase representation as $X(n) = [X_0(n) X_1(n) \dots X_{N-1}(n)]^T$. The data vector $X(n)$ is modulated by an inverse discrete Fourier transform (IDFT) into an OFDM symbol vector $x(n)$. A length G cyclic extension of the IDFT output is added to $x(n)$ as a guard interval. The resulting vector with the cyclic prefix is given by $x^*(n) = [x_{N-G}(n) \dots x_{N-1}(n) x_0(n) \dots x_{N-1}(n)]^T$. The vector $x^*(n)$ is transmitted through a frequency selective fading channel of order L , i.e., the channel impulse response $h(m, \tau) \neq 0$ for $\tau = L$ and $h(m, \tau) = 0$ for $\tau > L$.

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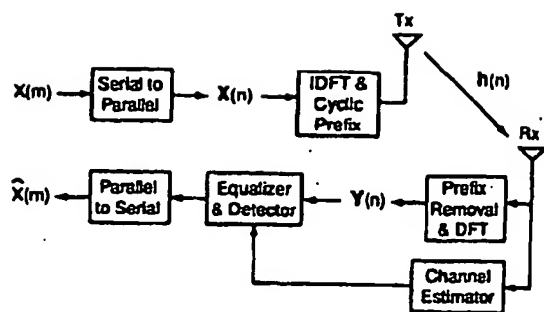


Fig. 1. Block diagram of a conventional OFDM system.

To avoid inter-block interference (IBI), the guard interval is chosen to satisfy $G \geq L$. Assuming the channel impulse response remains constant during the entire block interval, the received signal vector $y^s(n)$ is simply the convolution of $x^s(n)$ and the channel impulse response vector $h(n)$, i.e., $y^s(n) = x^s(n) * h(n)$. At the receiver, the guard interval is first removed from the received signal vector to form the vector $y(n) = [y_0^s(n) y_{G-1}^s(n) \dots y_{N-G-1}^s(n)]^T$. It can be shown that with $x^s(n)$ constructed as the cyclic extension of $x(n)$, the vector $y(n)$ is the cyclic convolution of $x(n)$ and $h(n)$. The demodulator performs an N -point discrete Fourier transform (DFT) on the vector $y(n)$ to yield the demodulated signal vector $Y(n)$. A well-known property of the DFT is that cyclic convolution in the time domain results in multiplication in the frequency domain. Thus, the demodulated signal vector is given by $Y(n) = \Lambda(n)X(n) + Z(n)$, where $\Lambda(n)$ is a diagonal matrix whose diagonal elements are the DFT of the channel impulse response $h(n)$ and $Z(n)$ is the DFT of the channel noise. Since $\Lambda(n)$ is diagonal, the subchannels are completely decoupled from each other, and $\Lambda_{k,k}(n)$ can be viewed as the complex channel gain of the k -th subcarrier. Thus, OFDM with cyclic prefix transforms a frequency selective fading channel into N perfectly flat fading subchannels on which orthogonal space-frequency transmitter diversity technique can be applied.

III. SF-OFDM TRANSMITTER DIVERSITY

The proposed two-branch SF-OFDM transmitter diversity system is an OFDM extension of the simple orthogonal transmitter diversity scheme first shown in [2]. The OFDM modulation with cyclic prefix allows the orthogonal transmitter diversity technique to work in frequency selective channels. A simplified block diagram for the proposed system is shown in Fig. 2.

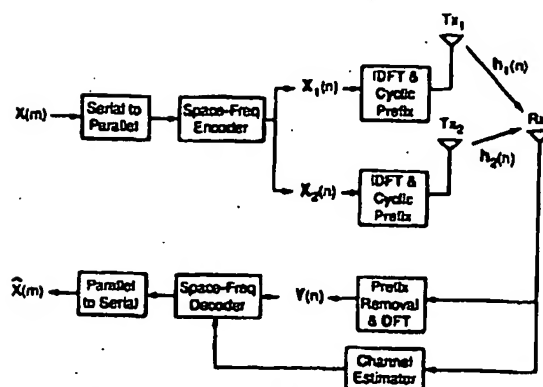


Fig. 2. Block diagram of the proposed two-branch space-frequency OFDM transmitter diversity system.

The data symbol vector $X(n)$ is coded into two vectors $X_1(n)$ and $X_2(n)$ by the space-frequency encoder block as

$$\begin{aligned} X_1(n) &= [X_0(n) - X_1^*(n) \dots X_{N-2}(n) - X_{N-1}^*(n)]^T \\ X_2(n) &= [X_1(n) \ X_0^*(n) \dots X_{N-1}(n) \ X_{N-2}^*(n)]^T \end{aligned} \quad (1)$$

During the block instant n , $X_1(n)$ is transmitted from the first base station Tx_1 , while $X_2(n)$ is transmitted simultaneously from the second base station Tx_2 .

The operations of the space-frequency encoder and decoder can best be described in terms of even and odd poly-phase component vectors. Let $X_e(n)$ and $X_o(n)$ be two length $N/2$ vectors denoting the even and odd component vectors of $X(n)$, i.e.,

$$\begin{aligned} X_e(n) &= [X_0(n) \ X_2(n) \dots X_{N-4}(n) \ X_{N-2}(n)]^T \\ X_o(n) &= [X_1(n) \ X_3(n) \dots X_{N-3}(n) \ X_{N-1}(n)]^T \end{aligned}$$

Similarly, $X_{1e}(n)$, $X_{1o}(n)$, $X_{2e}(n)$ and $X_{2o}(n)$ denote the even and odd component vectors of $X_1(n)$ and $X_2(n)$ respectively. Equation (1) can then be expressed in terms of the even and odd component vectors as

$$\begin{aligned} X_{1e}(n) &= X_e(n) \quad , \quad X_{1o}(n) = -X_o^*(n) \\ X_{2e}(n) &= X_o(n) \quad , \quad X_{2o}(n) = X_e^*(n) \end{aligned} \quad (2)$$

The equivalent space-frequency block code transmission matrix [3] is given by

$$G_1 = \begin{pmatrix} X_e & X_o \\ -X_o^* & X_e^* \end{pmatrix}$$

Let $\Lambda_1(n)$ and $\Lambda_2(n)$ be two diagonal matrices whose elements are the DFTs of the respective channel impulse responses, $h_1(n)$ and $h_2(n)$. The demodulated signal at the receiver is given by

$$Y(n) = A_{1,n}(n)X_1(n) + A_{2,n}(n)X_2(n) + Z(n) \quad (3)$$

or, equivalently, as

$$\begin{aligned} Y_1(n) &= A_{1,1,n}(n)X_{1,1}(n) + A_{1,2,n}(n)X_{2,1}(n) + Z_1(n) \\ Y_2(n) &= A_{2,1,n}(n)X_{1,1}(n) + A_{2,2,n}(n)X_{2,1}(n) + Z_2(n) \end{aligned} \quad (4)$$

Assuming the channel responses are known or can be estimated accurately at the receiver, the space-frequency decoder block constructs the decision estimate vector $\hat{X}(n)$ as

$$\begin{aligned} \hat{X}_1(n) &= A_{1,1,n}^*(n)Y_1(n) + A_{1,2,n}^*(n)Y_2(n) \\ \hat{X}_2(n) &= A_{2,1,n}^*(n)Y_1(n) - A_{2,2,n}^*(n)Y_2(n) \end{aligned} \quad (5)$$

The diversity performance of the proposed SF-OFDM transmitter diversity system can be analyzed by first substituting (2) into (4) to express the demodulated signal in terms of $X_1(n)$ and $X_2(n)$ as

$$\begin{aligned} Y_1(n) &= A_{1,1,n}(n)X_1(n) + A_{1,2,n}(n)X_2(n) + Z_1(n) \\ Y_2(n) &= -A_{1,2,n}(n)X_1(n) + A_{2,2,n}(n)X_2(n) + Z_2(n) \end{aligned} \quad (6)$$

Assuming the complex channel gains between adjacent subcarriers are approximately constant, i.e., $A_{1,1,n}(n) \approx A_{1,1,n}(n)$ and $A_{1,2,n}(n) \approx A_{1,2,n}(n)$, then substituting (6) into (5) results in

$$\begin{aligned} \hat{X}_1 &= (|A_{1,1,n}|^2 + |A_{1,2,n}|^2)X_1 + A_{1,1,n}^*Z_1 + A_{1,2,n}^*Z_2 \\ \hat{X}_2 &= (|A_{1,2,n}|^2 + |A_{2,2,n}|^2)X_2 + A_{1,2,n}^*Z_1 - A_{2,2,n}^*Z_2 \end{aligned} \quad (7)$$

where the implicit dependency on the block instant n has been omitted for brevity. Note that the decision variable equation (7) for the proposed SF-OFDM transmitter diversity scheme is similar in form to that of the optimal two-branch maximal ratio combining (MRC) receiver diversity system [7] and is exactly the same as that of the ST-OFDM transmitter diversity system in [5]. It is thus reasonable to expect the proposed SF-OFDM transmitter diversity system to have the same diversity performance as the previously reported ST-OFDM transmitter diversity system.

IV. PERFORMANCE OF SF-OFDM TRANSMITTER DIVERSITY

The bit error rate (BER) performance of the proposed two-branch SF-OFDM transmitter diversity system was verified by simulation. The simulation system used OFDM with cyclic prefix length set to the channel order L and 4-QAM on each subcarrier. The COST207 six-ray channel power delay profiles [8] were used throughout the simulations, and it was assumed that perfect channel estimation was available at the receiver. Simulation results for the SF-OFDM transmitter diversity system in a slow fading typical urban (TU) channel with maximum Doppler frequency $f_D = 10\text{Hz}$ are shown in Fig. 3.

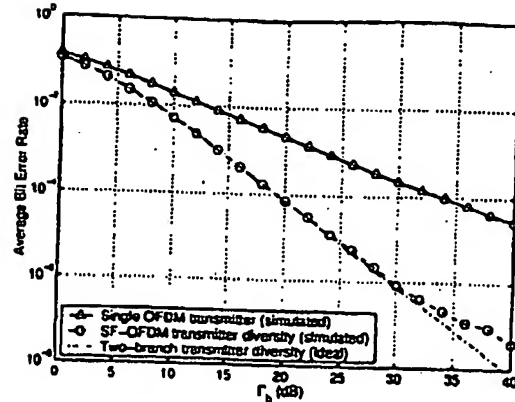


Fig. 3. SF-OFDM transmitter diversity in typical urban (TU) channel, $T_s = 2^{-20}\text{Sec.}$, $N = 512$ and $f_D = 10\text{Hz}$.

Simulation results show that the SF-OFDM transmitter diversity system significantly outperforms the conventional OFDM system. It provides about 5 dB of diversity gain at a bit error rate (BER) of 10^{-3} and about 15 dB of gain at a BER of 10^{-4} . Also shown in Fig. 3 is the theoretical performance curve of an ideal two-branch transmitter diversity system. The performance of the SF-OFDM transmitter diversity system very well up to a signal to noise ratio (SNR) of 30 dB. At higher SNR, the SF-OFDM transmitter diversity curve starts to exhibit an error floor, which is a well-known characteristic of all OFDM systems in Doppler spread channels [9].

For conventional OFDM systems, the BER error floor is mainly a function of the normalized Doppler frequency $f_D N T_s$. The BER error floor will degrade as $f_D N T_s$ increases [9]. With transmitter diversity, complex channel gain variation between successive OFDM blocks (in the ST-OFDM case) or between subcarriers (in the SF-OFDM case) can degrade the error floor of the diversity systems. The decision variables for ST-OFDM transmitter diversity are computed over two OFDM blocks [5], so it relies on the channel response to remain constant for two block periods $2NT_s$. With SF-OFDM transmitter diversity, the decision variables are completely determined within a single OFDM block, so it is reasonable to expect SF-OFDM to perform better than ST-OFDM in fast fading environments where $f_D N T_s$ is large. In Fig. 4 (a), the performance of the proposed SF-OFDM transmitter diversity technique is compared to that of the previously reported ST-OFDM transmitter diversity technique [5]. Simulation results show that SF-OFDM transmitter diversity significantly outperforms ST-OFDM transmitter diversity when the normalized Doppler frequency is large.

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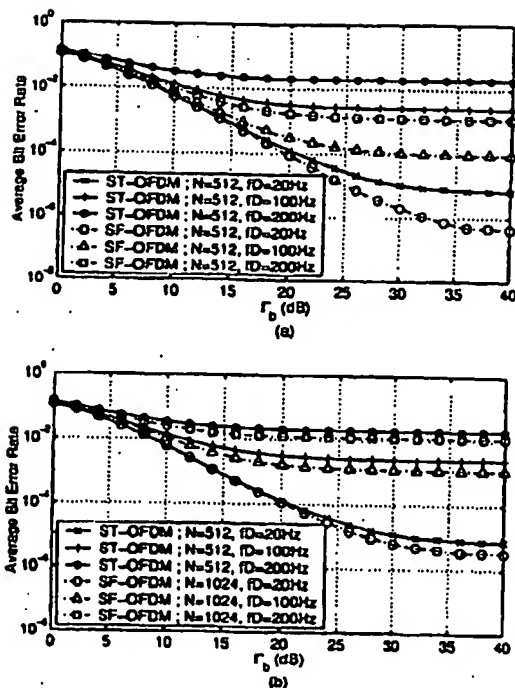


Fig. 4. Performance comparison of ST-OFDM and SF-OFDM transmitter diversity systems over the TU channel. (a) The same f_dNT , is used for both ST-OFDM and SF-OFDM. (b) f_dNT , is doubled for SF-OFDM.

To further illustrate the effectiveness of SF-OFDM in fast fading environments, the block size is doubled for the SF-OFDM system in Fig. 4 (b). Simulation results show that SF-OFDM still slightly outperforms ST-OFDM transmitter diversity even at twice the normalized Doppler frequency.

Besides its superior performance in fast fading environments, SF-OFDM transmitter diversity has other practical implementation advantages over the ST-OFDM approach, as well. Since the proposed SF-OFDM transmitter diversity scheme performs the decoding within one OFDM block, it only requires half of the decoder memory needed for the ST-OFDM system of the same block size. Similarly, the decoder latency for SF-OFDM is also half that of the ST-OFDM implementation.

Despite the number of advantages SF-OFDM transmitter diversity has over the ST-OFDM approach, there is one important parameter that requires careful consideration for SF-OFDM transmitter diversity systems. Recall the assumption used to derive (7) that the complex channel gain remains constant between adjacent subcarriers. Any significant

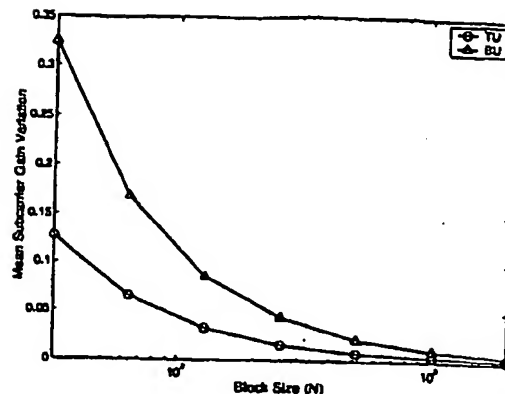


Fig. 5. Mean complex channel gain variation between adjacent subcarriers vs. block size N for typical urban (TU) and bad urban (BU) channel power delay profiles.

variation in complex channel gain between adjacent subcarriers will introduce error in the decoding process and will degrade the BER performance of the diversity system. The assumption of constant gain between subcarriers is, of course, only true for non-dispersive or flat channels. For frequency selective fading channels, the complex channel gain variation between subcarriers depends on the channel order L , the channel power delay profile, and the block size N . The gain variation between subcarriers will be worse for higher order channels and channels with larger root-mean-square (RMS) delay spreads. On the other hand, a larger block size reduces the gain variation between subcarriers because of the finer partitioning of the frequency selective channel.

Although an analytical expression for the subcarrier gain variation is not available, simulation of the complex channel gain of the subcarriers can provide a qualitative measure on how the gain variation is affected by the channel characteristics and block size. Fig. 5 shows the simulation results of the mean subcarrier gain variation as a function of the block size for the COST207 TU and bad urban (BU) channel power delay profiles. As expected, the gain variation between adjacent subcarriers is worse for small block sizes and for channels with poor power delay profiles, i.e., gain variation is worse for the BU channel because of its larger RMS delay spread. For $N = 32$, the mean gain variations are 0.13 for the TU channel and 0.33 for the BU channel. The gain variation becomes small quickly as N increases. For $N = 512$, the mean gain variations are reduced to 0.004 and 0.011 for the TU and BU channels, respectively.

Simulation results for an SF-OFDM transmitter diversity system with a block size of $N = 64$ in a slow fading BU channel is shown in Fig. 6. Although the normalized

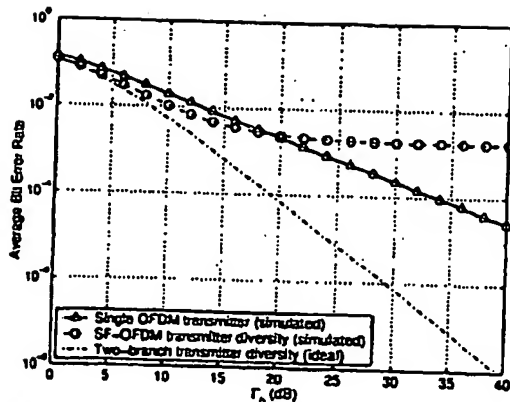


Fig. 6. SF-OFDM transmitter diversity over the bad urban (BU) channel with $T_s = 2^{-20}$ Sec., $N = 64$ and $f_d = 10$ Hz.

Doppler frequency is very small in this case, the BER is severely degraded because of excessive gain variation between subcarriers. Based on the results in Fig. 5 and Fig. 6, it can be concluded that a sufficiently large block size is essential for the implementation of SF-OFDM transmitter diversity systems. However, an excessively large block size is not required for SF-OFDM transmitter diversity to work effectively. For the symbol rate and channel characteristics used in the simulations for this study, a moderate block size of 512 is quite adequate as apparent from the results in Fig. 4 (a).

V. SUMMARY AND DISCUSSIONS

A simple two-branch SF-OFDM transmitter diversity technique for wireless communications over frequency selective fading channels has been presented. It has been shown that SF-OFDM is an efficient and effective transmitter diversity technique, especially for applications where the normalized Doppler frequency $f_d N T_s$ is large.

This paper has focused on two-branch diversity because of its simplicity and its unity coding rate. Higher order SF-OFDM transmitter diversity can be implemented in similar fashion along the frequency dimension. Unfortunately, higher order complex orthogonal block codes [3] all have less than unity coding rate, which results in a reduction in data throughput or an expansion in bandwidth in order to maintain the same data rate. Even if the coding rate loss is acceptable, it is not clear whether using higher order transmitter diversity directly or applying other error correction codes (ECC) on top of the second order transmitter diversity system will achieve better overall performance. The tradeoffs between higher order diversity and ECC is an interesting topic for further study. Also not addressed in this paper is the important issue of channel estimation in the SF-OFDM

transmitter diversity setting. A channel parameter estimation technique has been proposed for the ST-OFDM transmitter diversity system in [10]. It would be interesting to investigate if similar or better channel estimation approaches can be developed for SF-OFDM transmitter diversity systems.

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